12-1 Introduction to Rational Expressions

Simplify the rational expression. Identify any excluded values.

\[
\frac{2x + 8}{2x + 4} \quad \text{Rational expression}
\]

\[
\frac{2(x + 4)}{2(x + 2)} \quad \text{Factor the numerator and denominator.}
\]

\[
\frac{\frac{1}{2}(x + 4)}{\frac{1}{2}(x + 2)} \quad \text{Divide out a common factor.}
\]

\[
\frac{x + 4}{x + 2} \quad \text{Simplify.}
\]

The simplest form of \(\frac{2x + 8}{2x + 4}\) is \(\frac{x + 4}{x + 2}\).

To find the excluded values:

\[
2x + 4 = 0 \quad \text{Set the denominator of the original rational expression equal to 0.}
\]

\[
2(x + 2) = 0 \quad \text{Factor the denominator.}
\]

\[
2 = 0 \text{ or } x + 2 = 0 \quad \text{Use the Zero Product Property.}
\]

\[
x + 2 - 2 = 0 - 2 \quad \text{Solve for } x, \text{ using the Subtraction Property of Equality.}
\]

\[
x = -2
\]

So the excluded value is \(-2\) (or \(x \neq -2\)).

Simplify each rational expression. Identify any excluded values.

1. \(\frac{a + 4}{a - 4}\)
2. \(\frac{b + 6}{b - 3}\)
3. \(\frac{5c}{5c - 10}\)
4. \(\frac{8n}{8n + 32}\)

\[
\frac{a + 4}{a - 4}; a - 4 = 0 \quad \frac{b + 6}{b - 3}; b - 3 = 0
\]

\[
a = 4 \quad b = 3
\]

\[
a + 4 \neq 0; a \neq 4 \quad b + 6 \neq 3; b \neq 3
\]

5. \(\frac{2x + 9}{5x}\)
6. \(\frac{7y + 11}{11y}\)
7. \(\frac{12y + 24}{6y}\)
8. \(\frac{15u + 45}{3u}\)

\[
\frac{2x + 9}{5x}; 5x = 0 \quad \frac{7y + 11}{11y}; 11y = 0
\]

\[
x = 0 \quad y = 0
\]

\[
2x + 9 = 0; x \neq 0 \quad 7y + 11 = 0; y \neq 0
\]

9. \(\frac{8x^2}{4x^3 + 2x^2}\)
10. \(\frac{6b^2}{6b^3 + 2b^2}\)
11. \(\frac{10d^2 - 4d}{5d - 2}\)
12. \(\frac{9h^2 - 15h}{3h - 5}\)

\[
\frac{(8x^2)}{(4x^3 + 2x^2)} \neq 2^2 \quad \frac{6b^2}{6b^3 + 2b^2} \neq 2^2
\]

\[
\frac{4}{2x + 1}; 4x^3 + 2x^2 = 0; \quad \frac{3b + 1}{6b^3 + 2b^2}; 0; x = 0
\]

\[
x = 0; 2x + 1 = 0; \quad 3b + 1 = 0; b = -\frac{1}{3}
\]

\[
x = \frac{-1}{2}; \quad x \neq \frac{-1}{2}; \quad \frac{3}{3b + 1}; b \neq \frac{-1}{3}; b \neq 0
\]

\[
2d; d \neq \frac{2}{5}
\]

\[
3h; h \neq \frac{5}{3}
\]
Simplify. State the excluded value(s) of the variable(s).

13. \( \frac{x^2 - x}{x^3 - 7x^2 + 6x} \)
   
   The expression is undefined for \( x = 0 \) or \( x = 1 \).

14. \( \frac{y^2 - y}{y^3 + 3y^2 - 4y} \)
   
   Factors out common factors: \( y(y + 4)(y + 1) \)
   
   The expression is undefined for \( y = -4, 0, 1 \).

15. \( \frac{z^4 + z^3}{z^6 - z^3} \)
   
   Use the Zero Product Property: \( z^3 = 0; z - 1 = 0 \)
   
   The expression is undefined for \( z = 0, 1 \).

16. \( \frac{p^5 - p^4}{p^3 + p^2} \)
   
   Undefined for \( p = -1, 0 \).

17. \( \frac{ac + 11ac^2}{abc + 9abc^2} \)
   
   Undefined.

18. \( \frac{vw - 8vw^2}{uvw - 5uvw^2} \)
   
   Undefined.

Problem Solving

19. If both the length and width of a rectangle are tripled, what is the ratio of the area of the original rectangle to the area of the new rectangle?

   Area of original rectangle: \( A = lw \)
   
   Area of new rectangle: \( A = (3l)(3w) = 9lw \)
   
   \( \frac{9lw}{lw} = 9 \). The ratio of the areas is \( \frac{1}{9} \).

20. A store sells balloons for advertising. Which balloons with a 10-in. radius or a 12-in. radius?
   
   (Hint: \( SA = 4\pi r^2 \) and \( V = \frac{4}{3}\pi r^3 \))
   
   \( \frac{SA}{V} = \frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3}{r}; 10 \text{ in. ratio: } \frac{3}{10} \)
   
   12-in. ratio: \( \frac{3}{10} > \frac{1}{4} \)
   
   10-in. balloons have the greater ratio.

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21. Explain how to find the values where the expression \( \frac{1}{x^3 + 3x^2 - x - 3} \) is undefined.

   Solve \( x^3 + 3x^2 - x - 3 = 0 \) Group: \( (x^3 + 3x^2) + (-x - 3) = 0 \)
   
   Factor out common factors: \( x^2(x + 3) = 0 \)
   
   Use the Distributive Property: \( (x + 3)(x^2 - 1) = 0 \)
   
   Factor the difference of squares: \( (x + 3)(x - 1)(x + 1) = 0 \)
   
   Use the Zero Product Property: \( x + 3 = 0 \) or \( x - 1 = 0 \) or \( x + 1 = 0 \)
   
   Use the Addition and Subtraction Properties of Equality: \( x = -3 \) or \( x = 1 \) or \( x = -1 \).
   
   The expression is undefined for \( x = -3, x = -1, \) and \( x = 1 \).