Lesson 5-1, pages 116–119.

Determine if \( f(x) = x^3 - 1 \) represents a linear function.

Make a function table. Look at the rate of change.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-9</td>
</tr>
<tr>
<td>-1</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
</tbody>
</table>

The constant change of +1 in \( x \) does not correspond to a constant change in \( y \).

Because the change in \( y \) is not constant,

\( f(x) = x^3 - 1 \) is not a linear function.

Find the slope of the line.

\[
\text{Slope } (m) = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{y_2 - y_1}{x_2 - x_1}
\]

Substitute \((-2, 3)\) and \((1, 1)\) into the formula, and simplify.

\[
\frac{1 - 3}{1 - (-2)} = -\frac{2}{3}
\]

The slope is \(-\frac{2}{3}\); because the slope is negative, the line slants down from left to right.

Tell whether the relation represents a linear function. Explain why or why not. Find the slope of the line if it represents a linear function.

1. \begin{array}{c|c|c|c}
\hline
x & y & \text{change in } x \text{ constant: } +1 & \text{change in } y \text{ varies} \\
\hline
-2 & -6 & 16 - 1 = 15; 1 - 0 = 0 & \text{not a linear function} \\
-1 & 1 & 0 - 1 = -1; 1 - (-6) = 7 & \text{not a linear function} \\
0 & 0 & & \\
1 & 1 & & \\
2 & 16 & & \\
\hline
\end{array}

2. \begin{array}{c|c|c|c}
\hline
x & y & \text{change in } x \text{ constant: } +1 & \text{change in } y \text{ varies} \\
\hline
-2 & -32 & 16 - 1 = 15; 1 - 0 = 0 & \text{not a linear function} \\
-1 & -32 & 0 - 1 = -1; 1 - (-6) = 7 & \text{not a linear function} \\
0 & -1 & & \\
1 & -1 & & \\
2 & -32 & & \\
\hline
\end{array}

3. \begin{array}{c|c|c|c}
\hline
x & y & \text{change in } x \text{ constant} & \text{change in } y \text{ is constant} \\
\hline
-2 & -3 & & \\
-1 & -3 & & \\
0 & 1 & & \\
1 & 3 & & \\
2 & 3 & & \\
\hline
\end{array}

4. not a linear function

5. not a linear function

6. a linear function

7. a linear function

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Tell whether the relation represents a linear function. Explain why or why not. Find the slope of the line if it represents a linear function.

8. \( y = -4x + 8 \)  
9. \( y = -3x - 6 \)  
10. \( y = -2x^2 + 5 \)  
11. \( y = 3x^2 - 2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>16</td>
</tr>
<tr>
<td>-1</td>
<td>12</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

linear function  
constant rate: \( \frac{-4}{1} \)  
slope \((m) = -4\)

12. \( y = x - 2.5 \)  
13. \( y = -x - 3.9 \)  
14. \( y = \frac{2}{3}x - 1 \)  
15. \( y = \frac{1}{4}x + 1 \)

16. \( y = -\frac{2}{3}x - 1 \)  
17. \( y = -\frac{1}{6}x + 1 \)  
18. \( y = 8 \)  
19. \( x = -4 \)

20. \( y = -0.25x + 2 \)  
21. \( y = 0.125x - 2 \)  
22. \( y = -4x^3 \)  
23. \( y = \frac{x}{3} + 1 \)
### Answers for Algebra 1, Practice Book Lesson 5-1, page 110.

#### Linear Functions and Slope

**Table 9:**
- **x** | **y**
  - 2 | 0
  - 1 | -3
  0  | -6
  1  | -9
  2  | -12

**Linear Function:**
- Constant rate: \(-\frac{3}{1}\)
- Slope \((m) = -3\)

**Table 10:**
- **x** | **y**
  - 2 | -3
  - 1 | 3
  0  | 5
  1  | 3
  2  | -3

**Not a Linear Function:**
- No constant rate
- Slope \((m) = \text{undefined}\)

**Table 11:**
- **x** | **y**
  - 2 | 10
  - 1 | 1
  0  | -2
  1  | 1
  2  | 10

**Not a Linear Function:**
- No constant rate

**Table 12:**
- **x** | **y**
  - 2 | -4.5
  - 1 | -3.5
  0  | -2.5
  1  | -1.5
  2  | -0.5

**Linear Function:**
- Constant rate: \(\frac{1}{1}\)
- Slope \((m) = 1\)

**Table 13:**
- **x** | **y**
  - 2 | -1.9
  - 1 | -2.9
  0  | -3.9
  1  | -4.9
  2  | -5.9

**Linear Function:**
- Constant rate: \(-\frac{1}{1}\)
- Slope \((m) = -1\)

**Table 14:**
- **x** | **y**
  - 6 | -5
  - 3 | -3
  0  | -1
  3  | 1
  6  | 3

**Linear Function:**
- Constant rate: \(\frac{2}{3}\)
- Slope \((m) = \frac{2}{3}\)

**Table 15:**
- **x** | **y**
  - 4 | 0
  - 2 | 0.5
  0  | 1
  2  | 1.5
  4  | 2

**Linear Function:**
- Constant rate: \(\frac{1}{4}\)
- Slope \((m) = \frac{1}{4}\)

**Table 16:**
- **x** | **y**
  - 10 | 3
  - 5  | 1
  0   | 1
  5   | -3
  10  | -5

**Linear Function:**
- Constant rate: \(-\frac{2}{5}\)
- Slope \((m) = -\frac{2}{5}\)

**Table 17:**
- **x** | **y**
  - 12 | 3
  - 6  | 2
  0   | 1
  6   | 0
  12  | -1

**Linear Function:**
- Constant rate: \(-\frac{1}{6}\)
- Slope \((m) = -\frac{1}{6}\)

**Table 18:**
- **x** | **y**
  - 2  | 8
  - 1  | 8
  0   | 8
  1   | 8
  2   | 8

**Linear Function:**
- Constant rate: \(\frac{0}{1}\)
- Slope \((m) = 0\)

**Table 19:**
- **x** | **y**
  - 4  | -2
  - 4  | -1
  - 4  | 0
  - 4  | 1
  - 4  | 2

**Not a Function:**
- Constant rate: \(\frac{1}{0}\)
- Slope \((m) = \text{undefined}\)

**Table 20:**
- **x** | **y**
  - 4  | 3
  - 2  | 2.5
  0   | 2
  2   | 1.5
  4   | 1

**Linear Function:**
- Constant rate: \(-\frac{0.5}{2}\)
- Slope \((m) = -\frac{1}{4}\)

**Table 21:**
- **x** | **y**
  - 8  | -3
  - 4  | -2.5
  0   | -2
  4   | -1.5
  8   | -1

**Linear Function:**
- Constant rate: \(\frac{0.5}{4}\)
- Slope \((m) = \frac{1}{8}\)

**Table 22:**
- **x** | **y**
  - 2  | 32
  - 1  | 4
  0   | 0
  1   | -4
  2   | -32

**Not a Linear Function:**
- No constant rate

**Table 23:**
- **x** | **y**
  - 6  | -1
  - 3  | 0
  0   | 1
  3   | 2
  6   | 3

**Linear Function:**
- Constant rate: \(\frac{1}{3}\)
- Slope \((m) = \frac{1}{3}\)
Find the slope of the line for each graph.

24.  
\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 2}{3 - 0} = \frac{6}{3} = 2 \]  
The slope is 2.

25.  
\[ m = \frac{8 - 2}{1 - (-1)} = \frac{6}{2} = 3 \]  
The slope is 3.

26.  
\[ m = \frac{-4 - 2}{0 - (-2)} = \frac{-6}{2} = -3 \]  
The slope is -3.

27.  
\[ m = \frac{-6 - 4}{4 - 2} = \frac{-10}{2} = -5 \]  
The slope is -5.

Find the slope of the line that contains the given points. Describe the line.

28. 
\[ m = \frac{-4 - 0}{-2 - 4} = \frac{-4}{-6} = \frac{2}{3} \]  
The slope is \( \frac{2}{3} \).

29. 
\[ m = \frac{-2 - 4}{6 - (-4)} = \frac{-6}{10} = \frac{-3}{5} \]  
The slope is \( \frac{-3}{5} \).

30. 
The slope is 0.

31. 
The slope is undefined.

Find the slope of the line that contains the given points. Describe the line.

32. (3, 8) and (9, 6)  
\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 8}{9 - 3} = \frac{-2}{6} = -\frac{1}{3} \]  
slope = \( -\frac{1}{3} \); The line slants down from left to right.

33. (2, 7) and (11, 4)  
\[ m = \frac{7 - 4}{2 - 11} = \frac{3}{-9} = -\frac{1}{3} \]  
slope = \( -\frac{1}{3} \); The line slants down from left to right.

34. (8, -9) and (-6, -7)  
\[ m = \frac{-9 - (-7)}{-8 - (-6)} = \frac{-2}{-2} = 1 \]  
slope = 1; The line slants up from left to right.

35. (-11, -7) and (-8, -10)  
\[ m = \frac{-7 - (-10)}{-11 - (-8)} = \frac{3}{-3} = -1 \]  
slope = -1; The line slants down from left to right.

36. (6.3, 8) and (6.3, 12)  
\[ m = \frac{12 - 8}{6.3 - 6.3} = 0 \]  
slope is undefined; The line is vertical.

37. (-2, \( \frac{2}{3} \)) and (4, \( \frac{2}{3} \))  
\[ m = \frac{\frac{2}{3} - \frac{2}{3}}{5 - (-4)} = 0 \]  
slope = 0; The line is horizontal.

38. (3.4, -2.5) and (5.4, -7.5)  
\[ m = \frac{-7.5 - (-2.5)}{3.4 - 5.4} = \frac{5}{-2} = -\frac{5}{2} \]  
slope = \( -\frac{5}{2} \); The line slants down from left to right.

39. (1.8, -3.9) and (5.8, -6.9)  
\[ m = \frac{-6.9 - (-3.9)}{5.8 - 1.8} = \frac{-3}{4} \]  
slope = \( -\frac{3}{4} \); The line slants down from left to right.

40. (\( \frac{2}{3} \), \( \frac{2}{3} \)) and (4, 6)  
\[ m = \frac{\frac{2}{3} - \frac{2}{3}}{20 - 11} = \frac{0}{9} = 0 \]  
slope = 0; The line is horizontal.

41. (\( \frac{7}{3} \), \( \frac{4}{3} \)) and (\( \frac{1}{3} \), \( \frac{2}{3} \))  
\[ m = \frac{\frac{2}{3} - \frac{4}{3}}{\frac{1}{3} - \frac{7}{3}} = \frac{-2}{-6} = \frac{1}{3} \]  
slope = \( \frac{1}{3} \); The line slants up from left to right.

42. (2.7, 6.3) and (1, 8)  
\[ m = \frac{8 - 6.3}{1 - 2.7} = \frac{1.7}{-1.7} = -1 \]  
slope = -1; The line slants down from left to right.

43. (-4.1, 7.1) and (9, -6)  
\[ m = \frac{-6 - 7.1}{9 - (-4.1)} = \frac{-13.1}{13.1} = -1 \]  
slope = -1; The line slants down from left to right.
Solve. Show your work.

44. The graph shows the cost of Keri’s monthly calling plan on her cell phone.

![Keri's Monthly Calling Plan graph]

- a. What is the slope of this line? What does it mean?
  
  \[ m = \frac{1}{2}; \text{ It means that every additional minute that Keri uses her phone costs 50 cents.} \]

- b. This month, Keri used 28 additional minutes. How much was this month’s bill?
  
  \[ 40 + \frac{1}{2}(28) = 40 + 14 = 54; \text{ This month’s bill is $54.} \]

45. Environment According to data provided by the Permanent Service for Mean Sea Level (PSMSL), the global sea level has been rising since 1870. According to their data, the global sea level had risen about 1 cm by 1920 and about 12 cm by 1980.

- a. Using the rate for 1920 to 1980, at about what rate is the sea level rising?
  
  \[ \frac{12 - 1}{1980 - 1920} = \frac{11}{60} = 0.18 \text{ cm/yr} \]

- b. At this rate, by how much will the global sea level have risen from 1870 to 2101?
  
  \[ 2101 - 1980 = 121; 121(0.18) = 21.78 \text{ cm} \]
  
  The sea level will have risen 33.78 cm by 2101.

Problem Solving

46. One side of a roof has a slope of \( \frac{1}{4} \). The horizontal length of the roof is 48 ft. What is the height of the roof if the highest point is above the center of the horizontal length?

![Roof diagram]

\[ \text{slope} = \frac{24}{48}; x = 6 \]

The roof has a height of 6 ft.

47. A hill rises 80 ft vertically over a 15-ft horizontal distance. A nearby hill rises at the same rate but is 300 ft tall. If the hills’ highest points are above the center of the horizontal length of each hill, what is the horizontal length of the nearby hill?

\[ \text{slope} = \frac{80}{15} = \frac{300}{x}; 40x = 4500; x = 112.5 \]

The nearby hill has a height of 112.5 ft.

48. A line with a slope of \( \frac{7}{8} \) passes through the point \((-15, -13)\).

A different line with a slope of \(-1\) passes through the point \((4, -2)\).

At what point do the two lines cross?

Possible response: Make an organized list, and list points for both lines.

- line with slope of \( \frac{7}{8} \): \((-15, -13), (-7, -6), (1, 1), (9, 8), \ldots \)
- line with slope of \(-1\): \((4, -2), (3, -1), (2, 0), (1, 1), \ldots \)

The lines will cross at \((1, 1)\).
47. Hill 1     Hill 2

\[
\begin{align*}
\text{Hill 1} & \quad 80 \text{ ft} \\
15 \text{ ft} & \quad x \\
\text{Hill 2} & \quad 300 \text{ ft}
\end{align*}
\]