7-4 Model Binomial Multiplication

Name ______________________ Date ______________

Find each product using an area model.

(3x + 4)(2x + 1)

3x + 2 = 6x²
3x • 1 = 3x
4 • 2x = 8x
4 • 1 = 4

3x • 2x = 6x²
4 • 2x = 8x

Find the area of each region:
3x • 2x = 6x²
3x • 1 = 3x
4 • 2x = 8x
4 • 1 = 4

6x² + 3x + 8x + 4 ← Add the areas of the 4 regions.
6x² + 11x + 4 ← Combine like terms.

So (3x + 4)(2x + 1) = 6x² + 11x + 4

(5x – 3)(3x + 2)

(5x – 3)(5x – 3)(5x + 2)

To find (5x – 3)(3x + 2),
subtract (15x² + 10x) – (9x + 6).

(5x – 3)(3x + 2) = 15x² + 10x – 9x – 6
15x² + x – 6 ← Combine like terms.

Combine like terms.

So (5x – 3)(3x + 2) = 15x² + x – 6

Write the binomial expressions modeled by each diagram. Then find each product.

1. 

(4x + 5)(2x + 7)

4x • 2x = 8x²; 4x • 7 = 28x
5 • 2x = 10x; 5 • 7 = 35
8x² + 28x + 10x + 35
8x² + 38x + 35

2. 

(6x + 1)(7x + 3)

6x • 7x = 42x²; 6x • 3 = 18x
1 • 7x = 7x; 1 • 3 = 3
42x² + 18x + 7x + 3
42x² + 25x + 3

3. 

4. 

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3.

To find \((2x - 7)(3x - 4)\),
subtract \((2x)(3x) - (8x - 28 + 21x - 28 + 28)\).
So \((2x - 7)(3x - 4) = 6x^2 - 29x + 28\)

4.

To find \((4x - 5)(3x - 8)\),
subtract \((4x)(3x) - (32x - 40 + 15x - 40 + 40)\).
So \((4x - 5)(3x - 8) = 12x^2 - 47x + 40\)
Find the product. Sketch an area model on a separate sheet of paper.  
\[
5. (15 + 4)(15 - 4) \\
15 \cdot 15 = 225 \\
15 \cdot (-4) = -60 \\
4 \cdot 15 = 60 \\
4 \cdot (-4) = -16 \\
\text{225} - \text{60} + \text{60} - \text{16} = \text{209}
\]
\[
6. (30 + 3)(30 - 3) \\
30 \cdot 30 = 900 \\
30 \cdot (-3) = -90 \\
3 \cdot 30 = 90 \\
3 \cdot (-3) = -9 \\
900 - 90 + 90 - 9 = -891
\]
\[
7. (x + 4)(x + 7) \\
x \cdot x = x^2 \\
x \cdot 7 = 7x \\
4 \cdot x = 4x \\
4 \cdot 7 = 28 \\
x^2 + 7x + 4x + 28 = x^2 + 11x + 28
\]
\[
8. (x + 6)(x + 9) \\
x \cdot x = x^2 \\
x \cdot 9 = 9x \\
6 \cdot x = 6x \\
6 \cdot 9 = 54 \\
x^2 + 9x + 6x + 54 = x^2 + 15x + 54
\]
\[
9. (2x + 5)(5x - 7) \\
2x \cdot 5x = 10x^2 \\
2x \cdot (-7) = -14x \\
5 \cdot 5x = 25x \\
5 \cdot (-7) = -35 \\
10x^2 - 14x + 25x - 35 = 10x^2 + 11x - 35
\]
\[
10. (4x + 1)(6x - 1) \\
4x \cdot 6x = 24x^2 \\
4x \cdot (-1) = -4x \\
1 \cdot 6x = 6x \\
1 \cdot (-1) = -1 \\
24x^2 - 4x + 6x - 1 = 24x^2 + 2x - 1
\]
\[
11. (2x - 3)(3x - 2) \\
2x \cdot 3x = 6x^2 \\
2x \cdot (-2) = -4x \\
-3 \cdot 3x = -9x \\
-3 \cdot (-2) = 6 \\
6x^2 - 4x - 9x + 6 = 6x^2 - 13x + 6
\]
\[
12. (4x - 5)(5x - 4) \\
4x \cdot 5x = 20x^2 \\
4x \cdot (-4) = -16x \\
-5 \cdot 5x = -25x \\
-5 \cdot (-4) = 20 \\
20x^2 - 16x - 25x + 20 = 20x^2 - 41x + 20
\]
\[
13. (8x - 3)(5x + 2) \\
8x \cdot 5x = 40x^2 \\
8x \cdot 2 = 16x \\
-3 \cdot 5x = -15x \\
-3 \cdot 2 = -6 \\
40x^2 + 16x - 15x - 6 = 40x^2 + x - 6
\]
\[
14. (7x - 2)(10x + 3) \\
7x \cdot 10x = 70x^2 \\
7x \cdot 3 = 21x \\
-2 \cdot 10x = -20x \\
-2 \cdot 3 = -6 \\
70x^2 + 21x - 20x - 6 = 70x^2 + x - 6
\]
\[
15. (5x - 7)(5x + 7) \\
5x \cdot 5x = 25x^2 \\
5x \cdot 7 = 35x \\
-7 \cdot 5x = -35x \\
-7 \cdot 7 = -49 \\
25x^2 + 35x - 35x - 49 = 25x^2 - 49
\]
\[
16. (6x + 1)(6x - 1) \\
6x \cdot 6x = 36x^2 \\
6x \cdot (-1) = -6x \\
1 \cdot 6x = 6x \\
1 \cdot (-1) = -1 \\
36x^2 - 6x + 6x - 1 = 36x^2 - 1
\]

**Problem Solving**

17. A rectangular field has dimensions 3x + 9 by 5x + 11. In the center of the field is a square with side x + 1. What is the area inside the rectangle and outside the square?

\[
A = (3x + 9)(5x + 11) - [(x + 1)(x + 1)] \\
A = (15x^2 + 78x + 99) - (x^2 + 2x + 1) \\
A = 14x^2 + 76x + 98; \text{ The area inside the rectangle and outside the square is } 14x^2 + 76x + 98.
\]

18. A circle with diameter 8x + 10 has a square inside of it with side x + 2. What is the area inside the circle and outside the square?

\[
\text{Radius} = 4x + 5; \text{ area of circle: } \pi r^2 = \pi (4x + 5)(4x + 5) \\
\text{Area} = \pi (16x^2 + 40x + 25); \\
\text{area of square} = (x + 2)(x + 2) \\
\text{Area} = (x^2 + 4x + 4); \\
\text{area} = \pi (16x^2 + 40x + 25) - (x^2 + 4x + 4) \\
\text{The area inside the circle and outside the square is } (16\pi x^2 - x^2) + (40\pi x - 4x) + (25\pi - 4).
\]

**Critical Thinking**

19. If x \( \bullet \) x represents a square with area \( x^2 \), then what geometric figure does \( x \bullet x \bullet x \) represent?

The figure is a cube with dimensions x by x by x and a volume of \( x^3 \).